The Averaged Null Energy Condition for General Quantum Field Theories in Two Dimensions

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Dedicated to the memory of Klaus Baumann

Abstract. It is shown that the averaged null energy condition is fulfilled for a dense, translationally invariant set of vector states in any local quantum field theory in two-dimensional Minkowski spacetime whenever the theory has a mass gap and possesses an energy-momentum tensor. The latter is assumed to be a Wightman field which is local relative to the observables, generates locally the translations, is divergence-free, and energetically bounded. Thus the averaged null energy condition can be deduced from completely generic, standard assumptions for general quantum field theory in two-dimensional flat spacetime.

1 Introduction

The averaged null energy condition (ANEC, for short) has attracted some interest during the past several years as a possible candidate for a stability condition in semiclassical gravity. In its simplest form, this condition requires that in quantum field theory (on any spacetime manifold) the integral of the expectation value, $\langle T_{\mu\nu} \rangle$, of the energy-momentum tensor in any physical state, along any complete, lightlike geodesic γ is always non-negative:

$$\int_{-\infty}^{\infty} \langle T_{\mu\nu}(\gamma(s)) \rangle k^\mu k^\nu \, ds \geq 0 \,,$$

where s is an affine parameter and k^μ the (parallelly propagated) tangent of γ. (For a formulation not requiring the existence of the integral, see below.)

We shall briefly indicate the origin and development of this condition, however, we are not attempting to properly review this area of research and refer the reader to the articles [8, 24, 25, 10, 23] for further discussion and additional references.

In the theory of classical gravity, one central object of study is the behaviour of solutions to Einstein's equations,

$$G_{\mu\nu}(x) = 8\pi T_{\mu\nu}(x)\,,$$

for classical matter described by the energy-momentum tensor $T_{\mu\nu}$. There are important results asserting that a certain qualitative behaviour of these solutions must necessarily occur, within a broad range of initial conditions, as soon as certain stability requirements are imposed on $T_{\mu\nu}$. It is significant that such qualitative behaviour typically reflects a stability of causality, i.e. an initially causally well-behaved spacetime will not end up to develop, e.g., closed timelike curves. Most prominent among those results are the singularity theorems [13, 21]; the typical stability requirements in this context are the null energy condition,

$$T_{\mu\nu}(x)k^\mu k^\nu \geq 0 \tag{1.1}$$

for all lightlike vectors k^μ at any point x in spacetime, or the weak energy condition, where (1.1) is to hold for all causal vectors k^μ at any point x, and related variants, like the strong energy condition or the dominant energy condition, cf. [13, 21]. The common feature of these conditions is that they impose a local (even pointlike) positivity constraint like in eq. (1.1) on the energy-momentum tensor. For energy-momentum tensors of phenomenological models for classical matter, such local positivity constraints have largely been found to be physically realistic. In contrast, it is known that, under very general hypotheses, similar local positivity constraints cannot hold for the expectation values of the energy-momentum tensor $\langle\psi, T_{\mu\nu}(x)\psi\rangle$ of a quantum field in Minkowski-spacetime for a dense set of state vectors ψ [7].

Now, in semiclassical gravity, one investigates the semiclassical Einstein equation

$$G_{\mu\nu}(x) = 8\pi\langle T_{\mu\nu}(x)\rangle \tag{1.2}$$

where $\langle T_{\mu\nu}(x)\rangle$ is the expectation value of the energy-momentum tensor in a physical state of a quantum field propagating in a classical background spacetime whose Einstein tensor is $G_{\mu\nu}$. The question arises if there is a realistic replacement for the local positivity constraints on $\langle T_{\mu\nu}(x)\rangle$ leading to similar implications, i.e. the necessity of a certain, causally stable behaviour of solutions to (1.2) to occur. And in fact, candidates for such replacements have been found. In [19] it was observed that nonlocal, "averaged" versions of the local positivity constraints on the classical energy-momentum tensor still lead to essentially the same singularity theorems which result from imposing local positivity constraints (see also [5, 3, 17] for discussion and further results). The "averaged" refers to integrating the energy-momentum tensor along causal geodesics. The condition used in [19] is that

$$\int_{-\infty}^{\infty} \big(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T^\sigma{}_\sigma\big)(\gamma(s))\, k^\mu k^\nu\, ds \geq 0$$

for any complete causal geodesic with affine parameter s and tangent k^μ; $g_{\mu\nu}$ is the spacetime metric. This is referred to as averaged strong energy condition. In [17] it was shown that an averaged null energy condition for certain half-complete geodesics, i.e. essentially

$$\liminf_{r\to\infty} \int_0^r T_{\mu\nu}(\gamma(s))\, k^\mu k^\nu \, ds \geq 0 \tag{1.3}$$

for all lightlike geodesics γ with affine parameter s and tangent k^μ emanating at $s = 0$ from a closed trapped surface, implies singularity theorems. Moreover, it is proved in [3] that singularity theorems are implied by ANEC, roughly,

$$\liminf_{r_\pm\to\infty} \int_{-r_-}^{r_+} T_{\mu\nu}(\gamma(s))\, k^\mu k^\nu \, ds \geq 0 \tag{1.4}$$

for all complete lightlike geodesics with affine parameter s and tangent k^μ. (The precise formulations in [17] and [3] are slightly different from ours in (1.3) and (1.4). The reader is referred to these references for the technical details. The significant point is that the averaged energy conditions don't assume that the integrals converge, nor that they are bounded above.) It was also shown in [15] and [11] that the averaged null energy conditions (1.3) and (1.4), respectively, prevent the occurence of traversable wormholes in solutions to Einstein's equations.

In the light of these findings, an interesting issue is whether such averaged energy (positivity) conditions are fulfilled for the expectation values of the energy-momentum tensor for (suitable) states in quantum field theories. There have been several works dealing with this question and we continue by summarizing, however briefly, the results found so far. To fix our terminology, we say that a state ω of a quantum field theory on some background spacetime fulfills the ANEC (resp., AWEC = averaged weak energy condition) if the expectation value $\langle T_{\mu\nu}(x)\rangle_\omega$ of the energy-momentum tensor exists in this state as a (smooth) function of all x in spacetime, and if

$$\liminf_{r_\pm\to\infty} \int_{-r_-}^{r_+} \langle T_{\mu\nu}(\gamma(s))\rangle_\omega\, k^\mu k^\nu \, ds \geq 0 \tag{1.5}$$

holds for all complete lightlike (resp., timelike) geodesics γ with affine parameter s and tangent k^μ. (However, it should be noted that in some references slightly different formulations are used.)

In [14] it is shown that ANEC and AWEC are fulfilled for the free scalar field in n-dimensional Minkowski spacetime for states which are bounded in particle number and energy. It was also found in this work that AWEC is violated in some states of the free scalar field on a static, spatially closed two-dimensional spacetime.

The work [9] establishes ANEC for states bounded in particle number and energy of the free electromagnetic field in four-dimensional Minkowski spacetime.

In the article [23] it is shown that ANEC holds for all Hadamard states of the massless free scalar field in any two-dimensional globally hyperbolic spacetime, and for all Hadamard states of the massive free scalar field on two-dimensional Minkowski spacetime. Moreover it is proved that ANEC holds for the massive and massless

free scalar fields fulfilling some additional condition (implying that the limit $r_\pm \to \infty$ in (1.5) exists) in four-dimensional flat spacetime. In that work there appears also an argument indicating that ANEC cannot be expected to hold in general for the massless free scalar field on all four-dimensional curved spacetimes. Conditions implying that ANEC and AWEC will fail to hold generally for a large class of curved spacetimes are given in [20]. In [24, 25] it has therefore been suggested that the original formulation of ANEC should be altered via replacing the integrand of (1.5) by

$$\langle T_{\mu\nu}(\gamma(s))\rangle_\omega - D_{\mu\nu}(\gamma(s))$$

where $D_{\mu\nu}(x)$ is some state-independent tensor, e.g. the expectation value of the energy-momentum tensor in some reference state (like the vacuum in flat spacetime) or some quantity locally constructed from curvature terms. Such formulation of ANEC has been termed "difference inequality". Results in [24, 25] and [10] (cf. also [8]) indicate that such difference inequalities may have a better chance to hold generally in curved spacetime. We refer to the references for further discussion.

At any rate, investigations about the validity of ANEC (or difference inequalities) so far have been limited to the consideration of free fields only. The proofs of ANEC presented up to now strongly rely either on the fact that the quantum field obeys a linear hyperbolic equation of motion, or on the explicit form of the Wick-ordered energy-momentum tensor operator as bilinear expression in annihilation and creation operators in Fockspace. This is clearly unsatisfactory if one wishes to assess the general validity of ANEC in quantum field theory (say, in flat spacetime). Moreover, one would like to understand the connection of ANEC to the standard stability requirement in general quantum field theory, i.e. the spectrum condition and existence of a vacuum state.

In the present work, we make a first attempt towards clarifying the status of ANEC in general quantum field theory. We shall consider a general quantum field theory on two-dimensional Minkowski spacetime obeying the usual assumptions like locality, translation covariance, spectrum condition with mass gap and existence of a unique vacuum. Furthermore we assume that such a theory possesses an energy-momentum tensor, which is essentially supposed to be a Wightman field (operator valued distribution) characterized by being local relative to the observables, divergence-free, generating locally the translations, and fulfilling an energy-bound. The precise assumptions are given in Section 2.1. Comments on these assumptions and some well-known consequences (needed later) appear in Section 2.2. In Section 2.3 we prove that ANEC is fulfilled for a dense, translationally invariant set of vector states of any quantum field theory in two-dimensional Minkowski-spacetime fulfilling the general assumptions of Section 2.1. In Section 3 we show that ANEC will in general fail to hold if the integral averaging is carried out only along a lightlike geodesic half-line as in (1.3). This is of course expected in view of locality and the Reeh-Schlieder property. Some concluding remarks appear in Section 4.

We have opted to stage our discussion in the framework of the operator-algebraic approach to local quantum field theory (cf. [12, 1]) since this makes the structures involved in the argument particularly transparent. One could also obtain similar results working entirely in the setting of Wightman fields [18].

2 ANEC in quantum field theory on two-dimensional Minkowski spacetime

2.1 Assumptions

Our discussion of the ANEC in general quantum field theory on two-dimensional Minkowski spacetime begins by formulating the relevant assumptions.

Notation. Two-dimensional Minkowski-spacetime will be identified, as usual, with $\mathbb{R}^2$ with metric $(\eta_{\mu\nu}) = \mathrm{diag}(1,-1)$. The open forward lightcone is the set $V_+ := \{x \in \mathbb{R}^2 : (x^0)^2 - (x^1)^2 > 0,\ x^0 > 0\}$, the open backward lightcone is $V_- := -V_+$. The causal complement, $\mathcal{O}^\perp$, of a set $\mathcal{O} \subset \mathbb{R}^2$ is the largest open complement of the union of all sets $(V_+ \cup V_-) + x$, $x \in \mathcal{O}$. A *double cone* is a set of the form $\mathcal{O}_I := (S \backslash I)^\perp$ where S is any spacelike line in $\mathbb{R}^2$ (a spacelike hypersurface) and I any finite open subinterval of S. Any double cone is of the form $\mathcal{O} = (V_+ + y) \cap (V_- + x)$ for pairs of points $x, y \in \mathbb{R}^2$ with $x \in V_+ + y$. A *wedge region* is of the form $W = L(W_R)$ for any Poincaré transformation L where W_R is the right wedge, $W_R := \{(x^0, x^2) \in \mathbb{R}^2 : 0 < x^1,\ |x^0| < x^1\}$.

There will often appear the following special elements in $\mathbb{R}^2$:

$$e_0 := \binom{1}{0}\,, \quad e_1 := \binom{0}{1}\,, \quad e_+ := \tfrac{1}{\sqrt{2}}(e_0 + e_1)\,, \quad e_- := \tfrac{1}{\sqrt{2}}(e_0 - e_1)\,.$$

The summation convention is used throughout.

We shall now define what we mean by a quantum field theory with an energy-momentum tensor on two-dimensional Minkowski-spacetime: This is described in terms of a collection of objects $\{\mathcal{H}, \mathcal{A}, U, \Omega, T_{\mu\nu}\}$ whose properties are assumed to be as follows:

(i) $\mathcal{H}$ is a Hilbertspace, and there is a map $\mathcal{O} \mapsto \mathcal{A}(\mathcal{O})$ assigning to each double cone $\mathcal{O}$ in $\mathbb{R}^2$ a von Neumann algebra in $\mathcal{B}(\mathcal{H})$, with the properties:
$\tilde{\mathcal{O}} \subset \mathcal{O} \quad \Rightarrow \quad \mathcal{A}(\tilde{\mathcal{O}}) \subset \mathcal{A}(\mathcal{O})$ (isotony),
$\tilde{\mathcal{O}} \subset \mathcal{O}^\perp \quad \Rightarrow \quad \mathcal{A}(\tilde{\mathcal{O}}) \subset \mathcal{A}(\mathcal{O})'$ (locality). [1]

(ii) There is a weakly continuous representation $\mathbb{R}^2 \ni a \mapsto U(a)$ of the two-dimensional translation group by unitary operators on $\mathcal{H}$, fulfilling for all double cones $\mathcal{O}$,

$$U(a)\mathcal{A}(\mathcal{O})U(a)^* = \mathcal{A}(\mathcal{O} + a)\,, \quad a \in \mathbb{R}^2 \quad \text{(covariance)}.$$

(iii) There is an up to a phase unique unit vector $\Omega \in \mathcal{H}$ which is left invariant by the unitary group $U(a)$, $a \in \mathbb{R}^2$ (existence of a unique vacuum).

(iv) Denote by $P = (P_0, P_1)$ the generator of $U(a)$, $a \in \mathbb{R}^2$, i.e. $U(a) = \mathrm{e}^{iP_\mu a^\mu}$. Its spectrum fulfills

[1] Recall that $\mathcal{A}(\mathcal{O})'$ is the commutant of $\mathcal{A}(\mathcal{O})$, i.e. the algebra formed by all operators in $\mathcal{B}(\mathcal{H})$ that commute with every element in $\mathcal{A}(\mathcal{O})$.

$$\mathrm{sp}(P) \subset \{0\} \cup \{(p_0,p_1) \in \mathbb{R}^2 : (p_0)^2 - (p_1)^2 \geq m > 0,\ p_0 > 0\}$$

for some fixed $m > 0$ (spectrum condition with mass gap).

(v) The vacuum vector Ω is cyclic for union of the local von Neumann algebras $\bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$, i.e. the set $\bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O})\Omega$ is dense in $\mathcal{H}$ (cyclicity of the vacuum).

(vi) We denote by $\mathcal{A}_\infty$ the $*$-subalgebra of $\mathcal{B}(\mathcal{H})$ generated by all operators A of the form

$$A = \int h(a)\, U(a) B U(a)^*\, d^2a$$

for $h \in C_0^\infty(\mathbb{R}^2)$ and $B \in \bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$, and define:

$$\mathcal{A}_\infty(\mathcal{O}) := \mathcal{A}_\infty \cap \mathcal{A}(\mathcal{O})\,.$$

The energy-momentum tensor, $T_{\mu\nu}$, $\nu,\mu = 1,2$, is a set of operator valued distributions; more precisely, there is a dense domain $D \subset \mathcal{H}$, with $U(a)D \subset D$, $a \in \mathbb{R}^2$, and $\mathcal{A}_\infty\Omega \subset D$, so that for each $f \in C_0^\infty(\mathbb{R}^2)$, $T_{\mu\nu}(f)$ is a closable operator on D with $T_{\mu\nu}(\overline{f}) \subset T_{\mu\nu}(f)^*$. For each $\psi, \psi' \in D$, the map

$$C_0^\infty(\mathbb{R}^2) \ni f \mapsto \langle \psi, T_{\mu\nu}(f)\psi'\rangle$$

is a distribution in $\mathcal{D}'(\mathbb{R}^2)$.

(vii) Translation-covariance holds:

$$U(a)T_{\mu\nu}(f)U(a)^* = T_{\mu\nu}(f_a)\,, \quad f \in C_0^\infty(\mathbb{R}^2),\ a \in \mathbb{R}^2\,,$$

with $f_a(x) := f(x-a)$. Moreover, $T_{\mu\nu}$ has vanishing vacuum-expectation value:

$$\langle \Omega, T_{\mu\nu}(f)\Omega\rangle = 0\,, \quad f \in C_0^\infty(\mathbb{R}^2)\,.$$

(viii) $T_{\mu\nu}$ is local on the vacuum: $\langle A\Omega, [T_{\mu\nu}(f), B]\Omega\rangle = 0$
for all $A \in \mathcal{A}_\infty$, $B \in \mathcal{A}_\infty(\mathcal{O})$ and $f \in C_0^\infty(\mathcal{O}^\perp)$.

(ix) $T_{\mu\nu}$ is divergence-free on the vacuum:

$$\langle A\Omega, [T_{\mu\nu}(\partial^\mu f), B]\Omega\rangle = 0\,, \quad A, B \in \mathcal{A}_\infty\,.$$

(x) $T_{\mu\nu}$ generates (locally) the translations on the vacuum: Let S be the $x^0 = 0$ hyperplane (= spacelike line) with unit normal vector e_0. Whenever $a, b \in C_0^\infty(\mathbb{R})$ are any two non-negative functions with the properties
$a(x^0) = 0$ outside of some x^0-interval $(-\varepsilon_a, \varepsilon_a)$ and $\int a(x^0)\,dx^0 = 1$,
$b(x^1) = 1$ on an open x^1-interval $(\xi_b - \varepsilon_a - \delta_b, \xi_b + \varepsilon_a + \delta_b)$,
where $\varepsilon_a, \delta_b > 0$, $\xi_b \in \mathbb{R}$,
we require that, upon setting $\chi(x^0,x^1) := a(x^0)b(x^1)$, there holds

$$\begin{aligned} \langle A\Omega, [T_{\mu\nu}(\chi), B]\Omega\rangle e_0^\mu &= \langle A\Omega, [P_\nu, B]\Omega\rangle \\ &= \langle A\Omega, P_\nu B\Omega\rangle \end{aligned}$$

for all $A \in \mathcal{A}_\infty$ and all $B \in \mathcal{A}_\infty(\mathcal{O}_I)$ with $I = (\xi_b - \delta_b, \xi_b + \delta_b)$.

(xi) Energy bounds for $T_{\mu\nu}$: There is a pair of numbers $c, \ell > 0$ such that $(1+P_0)^{-\ell} T_{\mu\nu}(f)(1+P_0)^{-\ell}$ is for each $f \in C_0^\infty(\mathbb{R}^2)$ a bounded operator whose operator norm satisfies the estimate

$$|| (1+P_0)^{-\ell} T_{\mu\nu}(f)(1+P_0)^{-\ell} || \leq c \, || f ||_{L^1} \,, \quad f \in C_0^\infty(\mathbb{R}^2) \,.$$

2.2 Comments and some implications

The conditions (i)–(v) imply that we are considering a translation-covariant quantum field theory in a vacuum representation with mass gap, in operator algebraic formulation. These conditions are quite standard; the selfadjoint elements in $\mathcal{A}(\mathcal{O})$ are viewed as observables of the theory localized in the spacetime region $\mathcal{O}$, cf. [12] for further discussion.

Note that (v) and uniqueness of the vacuum vector imply irreducibility of the observable algebra, i.e. $(\bigcup_{\mathcal{O}} \mathcal{A}(\mathcal{O}))' = \mathbb{C}\,1$. Note also that locality, spectrum conditon and (v) imply the Reeh-Schlieder property of the algebras associated with wedge-regions W, defined as $\mathcal{A}(W) := (\bigcup_{\mathcal{O} \subset W} \mathcal{A}(\mathcal{O}))''$, i.e. the sets $\mathcal{A}(W)\Omega$ are dense in $\mathcal{H}$ for any wedge-region. It follows easily that then also the sets $\mathcal{A}_\infty(W)\Omega$ are dense in $\mathcal{H}$ for all wedge regions W. A slightly stronger assumption would be the Reeh-Schlieder property for the local algebras, i.e. that $\mathcal{A}(\mathcal{O})\Omega$ is dense in $\mathcal{H}$ for each double cone $\mathcal{O}$; this is the case when the local von Neumann algebras are weakly additive, as e.g. when there is a Wightman field generating the local algebras [16, 18]. Then it follows that $\mathcal{A}_\infty(\mathcal{O})\Omega$ is dense in $\mathcal{H}$. We will make such an assumption in Section 3.

The conditions (vi)–(xi) serve to characterize an energy-momentum tensor in the present abstract setting. Conditions (vi)–(viii) basically say that the energy-momentum tensor is a Wightman field which is local relative to the observables. Particularly important for the interpretation of $T_{\mu\nu}$ as an energy-momentum tensor are clearly (ix) and (x) expressing that, in a weak sense, $T_{\mu\nu}$ is divergence-free and generates locally the translations. Notice that on account of the assumed translation-covariance the condition formulated in (x) implies its validity for any translated copy $S + a$, $a \in \mathbb{R}^2$, of S in place of S. It is worth pointing out that we could have also taken for S any other spacelike hyperplane (= spacelike line) instead of the $x^0 = 0$ hyperplane, the proof of Theorem 2.5 below would then only involve changes in notation. Specializing to the $x^0 = 0$ hyperplane is thus just a matter of notational convenience.

Notice that for each $A \in \mathcal{A}_\infty$ one has $A\Omega \in \bigcap_{j \in \mathbb{N}} \mathrm{dom}\,(1+P_0)^j$. In view of the assumed energy bound, it actually follows that $\mathcal{A}_\infty\Omega$ is contained in the domain of $T_{\mu\nu}(f)$. (An assumption of this kind is clearly needed, otherwise it would be difficult to formulate that $T_{\mu\nu}(f)$ is local relative to the observables.) The energy bound (xi) has the simple interpretation that the local energy-momentum density integrated over a finite spacetime volume should be dominated by the total energy (or at least a sufficiently high moment of it). We mention as an aside that, if one assumes the domain D of $T_{\mu\nu}$ to coincide with the set $\bigcap_{j \in \mathbb{N}} \mathrm{dom}\,(1+P_0)^j$ and takes as testfunction-space the Schwartz-functions $\mathcal{S}(\mathbb{R}^2)$ instead of $C_0^\infty(\mathbb{R}^2)$, then this implies already an energy bound of the form as in (xi) [1, Prop. 12.4.10].

Finally, there arises the question if our assumptions regarding $T_{\mu\nu}$ are realistic. For free fields, the canonically constructed energy-momentum tensor fulfills the assumptions. (It fulfills, in particular, a linear energy bound, i.e. (xi) holds with $\ell = 1$.) While we have made no attempt to check this, it is to be expected that the quantum field models which have been constructed in two dimensions, like $P(\phi)_2$ or Yukawa$_2$, also comply with all of our assumptions.

The assumptions (i)–(xi) for a theory with energy-momentum tensor, $\{\mathcal{H}, \mathcal{A}, U, \Omega, T_{\mu\nu}\}$, are known to imply certain properties which will be used in deriving ANEC in the next section. Hence we subsequently collect these properties, mainly referring to the literature for proofs.

Proposition 2.1. [2, 6] *One has weak asymptotic lightlike clustering: For any lightlike $k \in \mathbb{R}^2\backslash\{0\}$ and any pair of vectors $\psi, \psi' \in \mathcal{H}$, it holds that*

$$\lim_{s\to\infty} \langle \psi, U(s \cdot k)\psi' \rangle = \langle \psi, \Omega \rangle \langle \Omega, \psi' \rangle \,. \tag{2.1}$$

Sketch of Proof: Let W_R be the right wedge region and $\mathcal{A}(W_R)$ the associated von Neumann algebra. Let Δ^{it}, $t \in \mathbb{R}$, be the modular group corresponding to $\mathcal{A}(W_R), \Omega$. Then a theorem by Borchers [2, Thm. II.9] establishes the relation

$$\Delta^{it} U(s \cdot e_+) \Delta^{-it} = U(\mathrm{e}^{-2\pi t} s \cdot e_+)$$

for all $t, s \in \mathbb{R}$. Consequently, one can apply the argument of Prop. I.1.3 in [6] to gain relation (2.1). We point out that the mass gap assumption enters in that argument.

Lemma 2.2. *Let $E := 1 - |\Omega\rangle\langle\Omega|$ be the projection orthogonal to the vacuum vector, and let $P_\pm := P_0 \pm P_1$. Let $\psi, \psi' \in \mathrm{dom}\,(P_0)$. Then there exist vectors $\psi_\pm \in \mathcal{H}$ such that*

$$\langle \psi, E\psi' \rangle = \langle \psi_\pm, EP_\pm \psi' \rangle \,.$$

Proof. From the mass-gap assumption we obtain

$$\frac{1}{|p_\pm|^2} \leq \frac{|p_0|^2}{m^2} \tag{2.2}$$

for all $p = (p_0, p_1) \in \mathrm{sp}(P)\backslash\{0\}$, where $p_\pm := p_0 \pm p_1$. We claim that the vectors $\psi_\pm := (P_\pm)^{-1} E\psi$ exist (in the sense of the functional calculus). Indeed, denoting the spectral measure of P by F, eqn. (2.2) implies

$$\begin{aligned} ||\, (P_\pm)^{-1} E\psi \,||^2 &= \int_{\mathrm{sp}(P)\backslash\{0\}} \frac{1}{|p_\pm|^2} \langle \psi, dF(p)\psi \rangle \\ &\leq \int_{\mathrm{sp}(P)\backslash\{0\}} \frac{|p_0|^2}{m^2} \langle \psi, dF(p)\psi \rangle \;\leq\; \frac{1}{m^2} ||\, P_0 \psi \,||^2 \,. \end{aligned}$$

Thus, by the functional calculus,

$$\langle \psi, E\psi' \rangle = \langle E\psi, E\psi' \rangle = \langle P_\pm (P_\pm)^{-1} E\psi, E\psi' \rangle = \langle \psi_\pm, EP_\pm \psi' \rangle \,,$$

where we used that E commutes with $P_\pm$. $\square$

Proposition 2.3. *Let $f \in C_0^\infty(\mathbb{R}^2)$ with $f \geq 0$, $\int f(x)\, d^2x = 1$, and define $f_{x,\lambda}(y) := \lambda^{-2} f(\lambda^{-1}(y-x))$ so that $f_{x,\lambda}$ approaches for $\lambda \to 0$ the delta-distribution concentrated at x. Then for each pair $A, B \in \mathcal{A}_\infty$, the limit*

$$\langle A\Omega, T_{\mu\nu}[x] B\Omega\rangle := \lim_{\lambda\to 0} \langle A\Omega, T_{\mu\nu}(f_{x,\lambda}) B\Omega\rangle$$

exists and defines a quadratic form on $\mathcal{A}_\infty\Omega \times \mathcal{A}_\infty\Omega$. Moreover,

(a) $\mathbb{R}^2 \ni x \mapsto \langle A\Omega, T_{\mu\nu}[x] B\Omega\rangle$ *is C^∞,*

(b) $\langle U(a)A\Omega, T_{\mu\nu}[x] U(a)B\Omega\rangle = \langle A\Omega, T_{\mu\nu}[x-a]\, B\Omega\rangle\,, \quad x, a \in \mathbb{R}^2$,

(c) $(1+P_0)^{-\ell} T_{\mu\nu}[x](1+P_0)^{-\ell} := \lim_{\lambda\to 0} (1+P_0)^{-\ell} T_{\mu\nu}(f_{x,\lambda})(1+P_0)^{-\ell}$ *is a bounded operator on $\mathcal{H}$,*

(d) $\langle A\Omega, [T_{\mu\nu}[x], B]\Omega\rangle = 0$ *for all $A \in \mathcal{A}_\infty$, $B \in \mathcal{A}_\infty(\mathcal{O})$ and $x \in \mathcal{O}^\perp$,*

(e) $\partial^\mu \langle A\Omega, [T_{\mu\nu}[x], B]\Omega\rangle = 0, \quad A, B \in \mathcal{A}_\infty$,

(f) $\int \langle A\Omega, [T_{\mu\nu}[x^1 e_1], B]\Omega\rangle e_0^\mu\, dx^1 = \langle A\Omega, P_\nu B\Omega\rangle, \quad A, B \in \mathcal{A}_\infty$.

This proposition is a fairly direct consequence of assumption (xi), see [1, Thm. 12.4.8] (cf. also references cited there). The commutator is defined as difference of quadratic forms:

$$\langle A\Omega, [T_{\mu\nu}[x], B]\Omega\rangle := \langle A\Omega, T_{\mu\nu}[x] B\Omega\rangle - \langle B^* A\Omega, T_{\mu\nu}[x]\Omega\rangle\,.$$

Observe that the integrand in (f) is supported on a finite interval because of (d). It should also be noted that $T_{\mu\nu}[x]$ will in general not exist as an operator.

Lemma 2.4. *Let W be a wedge region, $B \in \mathcal{A}_\infty$, and $j \in \mathbb{N}$. Then for each $\varepsilon > 0$ there is some $B_\varepsilon \in \mathcal{A}_\infty(W)$ such that*

$$||\,(1+P_0)^j (B - B_\varepsilon)\Omega\,|| < \varepsilon\,.$$

The proof can be given along similar lines as the proof of [1, Prop. 14.3.2]; we may therefore skip the details. The cyclicity of Ω for the algebras $\mathcal{A}_\infty(W)$ enters here. In combination with (b) and (c) of Prop. 2.3 one obtains as a simple corollary:

For each wedge region W, any $A, B \in \mathcal{A}_\infty$ and given $\varepsilon > 0$ there is some $B_\varepsilon \in \mathcal{A}_\infty(W)$ so that

$$|\langle A\Omega, T_{\mu\nu}[x](B - B_\varepsilon)\Omega\rangle| < \varepsilon \tag{2.3}$$

holds uniformly in $x \in \mathbb{R}^2$.

2.3 Main result

In the present section we state and prove our main result about ANEC in quantum field theory on two-dimensional Minkowski spacetime.

Theorem 2.5. *Let $\{\mathcal{H}, \mathcal{A}, U, \Omega, T_{\mu\nu}\}$ be a quantum field theory with energy-momentum tensor on two-dimensional Minkowski spacetime fulfilling the assumptions (i)–(xi) of Section 2.1.*

Let k be any non-zero lightlike vector in $\mathbb{R}^2$ and let $A, B \in \mathcal{A}_\infty$, $a \in \mathbb{R}^2$. Then it holds that

$$\lim_{r_\pm \to \infty} \int_{-r_-}^{r_+} \langle A\Omega, T_{\mu\nu}[s \cdot k + a]\, B\Omega \rangle k^\mu \, ds = \langle A\Omega, P_\nu B\Omega \rangle \,.$$

Corollary 2.6. *This implies the ANEC for all vector states induced by the dense, translation-invariant set of vectors $\{\psi = A\Omega\colon A \in \mathcal{A}_\infty\}$, corresponding to energetically strongly damped, local excitations of the vacuum:*

$$\lim_{r_\pm \to \infty} \int_{-r_-}^{r_+} \langle \psi, T_{\mu\nu}[s \cdot k + a]\, \psi \rangle k^\mu k^\nu \, ds = \langle \psi, k^\nu P_\nu \psi \rangle \geq 0$$

since k is lightlike and since the relativistic spectrum condition holds.

Proof. The proof proceeds in three simple steps. For simplicity of notation, we will give the proof only for the case $k = e_+$, the proof for $k = e_-$ is obtained by analogous arguments. In view of translation covariance, it suffices also to consider only the case $a = 0$.

1) We will first show that for all $C \in \mathcal{A}_\infty$

$$\lim_{s \to \pm\infty} \langle C\Omega, T_{\mu\nu}[s \cdot e_+]\Omega \rangle = 0 \,, \tag{2.4}$$

$$\lim_{r_\pm \to \infty} \int_{-r_-}^{r_+} \langle C\Omega, T_{\mu\nu}[s \cdot e_+]\Omega \rangle \, ds = 0 \,. \tag{2.5}$$

To this end, let

$$\begin{aligned} \psi &:= (1 + P_0)^{\ell+1} C\Omega \,, \\ \psi'_{\mu\nu} &:= (1 + P_0)^{-(\ell+1)} T_{\mu\nu}[0](1 + P_0)^{-\ell}\Omega \,. \end{aligned}$$

One can see from Prop. 2.3(c) that $\psi, \psi'_{\mu\nu} \in \mathrm{dom}\,(1 + P_0)$. Moreover, denoting by $E := 1 - |\Omega\rangle\langle\Omega|$ the projection orthogonal to the vacuum, we deduce, upon using assumption (vii) (implying $\langle \Omega, T_{\mu\nu}[x]\Omega \rangle = 0$) and Prop. 2.3(b) together with the fact that E commutes with $U(a)$, $a \in \mathbb{R}^2$, that

$$\langle C\Omega, T_{\mu\nu}[s \cdot e_+]\Omega \rangle = \langle \psi, E\, U(s \cdot e_+)\psi'_{\mu\nu} \rangle \,, \quad s \in \mathbb{R} \,.$$

Then relation (2.4) follows from weak asymptotic lightlike clustering, Prop. 2.1. Furthermore, by Lemma 2.2 it follows that there is a vector $\psi_+ \in \mathcal{H}$ so that

$$\langle \psi, E\, U(s \cdot e_+)\psi'_{\mu\nu} \rangle = \langle \psi_+, EP_+ U(s \cdot e_+)\psi'_{\mu\nu} \rangle = \frac{1}{i}\frac{d}{ds}\langle \psi_+, E\, U(s \cdot e_+)\psi'_{\mu\nu} \rangle \,.$$

Thus one obtains

$$
\begin{aligned}
\int_{-r_-}^{r_+} \langle C\Omega, T_{\mu\nu}[s\cdot e_+]\Omega\rangle \, ds &= \int_{-r_-}^{r_+} \frac{1}{i}\frac{d}{ds}\langle \psi_+, E\, U(s\cdot e_+)\psi'_{\mu\nu}\rangle \, ds \\
&= \frac{1}{i}\left(\langle \psi_+, E\, U(r_+\cdot e_+)\psi'_{\mu\nu}\rangle - \langle \psi_+, E\, U(r_-\cdot e_+)\psi'_{\mu\nu}\rangle\right)
\end{aligned}
$$

and the last expression tends to 0 in the limit $r_\pm \to \infty$ in view of weak asymptotic lightlike clustering, Prop. 2.1. This establishes relation (2.5).

2) Relation (2.5) shows, for any $A, B \in \mathcal{A}_\infty$,

$$
\lim_{r_\pm \to \infty} \left(\int_{-r_-}^{r_+} \langle A\Omega, T_{\mu\nu}[s\cdot e_+]B\Omega\rangle \, ds \; - \int_{-r_-}^{r_+} \langle A\Omega, [T_{\mu\nu}[s\cdot e_+], B]\Omega\rangle \, ds \right) = 0
$$

and hence, to prove the theorem, it suffices to demonstrate

$$
\lim_{r_\pm \to \infty} \int_{-r_-}^{r_+} \langle A\Omega, [T_{\mu\nu}[s\cdot e_+], B]\Omega\rangle e_+^\mu \, ds = \langle A\Omega, P_\nu B\Omega\rangle \,. \tag{2.6}
$$

To show this, we fix any $A, B \in \mathcal{A}_\infty$ and use the abbreviation

$$
\tau_{\mu\nu}(x) := \langle A\Omega, [T_{\mu\nu}[x], B]\Omega\rangle \,.
$$

Now we define two maps with values in $\mathbb{R}^2$,

$$
h_+(s,\rho) := s\cdot e_+ - \rho\cdot e_- \,, \quad h_-(s,\rho) := -s\cdot e_+ + \rho\cdot e_- \,, \quad s,\rho \geq 0 \,,
$$

and the two triangle-shaped regions

$$
\begin{aligned}
R_{+,r_+} &:= \{h_+(s,\rho) : 0 \leq s \leq r_+,\ 0 \leq \rho \leq s\} \,, \\
R_{-,r_-} &:= \{h_-(s,\rho) : 0 \leq s \leq r_-,\ 0 \leq \rho \leq s\} \,.
\end{aligned}
$$

The region R_{+,r_+} is bounded by the two lightlike line segments $L_+(r_+) := \{s\cdot e_+ : 0 \leq s \leq r_+\}$ and $H_+(r_+) := \{h_+(r_+,\rho) : 0 \leq \rho \leq r_+\}$, and by the spacelike line segment $S_+(r_+) := \{x^1 e_1 : 0 \leq x^1 \leq \sqrt{2}\, r_+\}$. Similarly, R_{-,r_-} is bounded by the line segments $L_-(r_-) := \{-s\cdot e_+ : 0 \leq s \leq r_-\}$, $H_-(r_-) := \{h_-(r_-,\rho) : 0 \leq \rho \leq r_-\}$, and $S_-(r_-) := \{-x^1 e_1 : 0 \leq x^1 \leq \sqrt{2}\, r_-\}$. (Cf. Figure 1.)

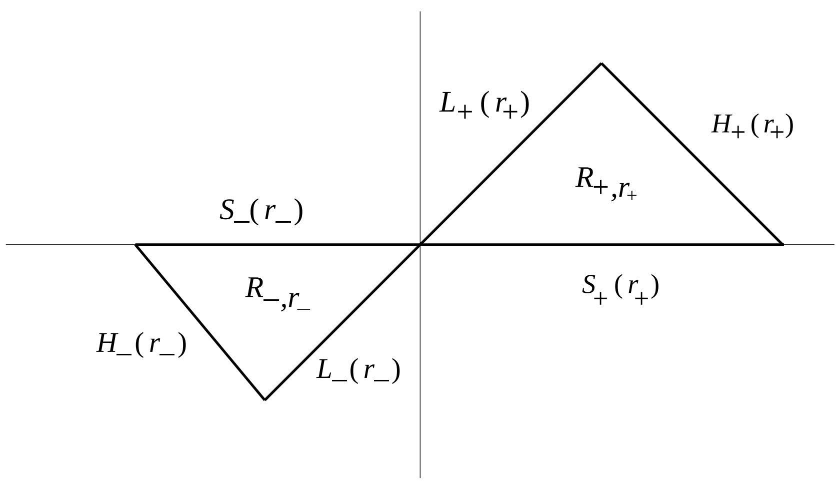

Figure 1. Sketch of the regions and bounding line segments described in the text.

Now we use $\partial^\mu \tau_{\mu\nu}(x) = 0$ and thus, applying Gauß' law to the region R_{+,r_+}, we convert the integral of $v^\mu = \tau^\mu{}_\nu$ paired with the outer normal along $L_+(r_+)$ into a sum of two integrals of v^μ paired with the inner normals along $H_+(r_+)$ and $S_+(r_+)$. Doing the same with respect to the region R_{-,r_-} (with the roles of inner and outer normals interchanged) yields, with the above parametrizations of the various line segments inserted,

$$\int_{-r_-}^{r_+} \tau_{\mu\nu}(s \cdot e_+) e_+^\mu \, ds = \int_{-\sqrt{2}r_-}^{\sqrt{2}r_+} \tau_{\mu\nu}(x^1 e_1) e_0^\mu \, dx^1 \tag{2.7}$$
$$- \int_0^{r_+} \tau_{\mu\nu}(h_+(r_+,\rho)) e_-^\mu \, d\rho \; - \int_0^{r_-} \tau_{\mu\nu}(h_-(r_-,\rho)) e_+^\mu \, d\rho \,.$$

In view of Prop. 2.3(d,f), we deduce that the first integral on the right hand side of (2.7) equals $\langle A\Omega, P_\nu B\Omega\rangle$ as soon as r_+ and r_- are large enough. This implies that (2.6), and hence the statement of the theorem, is proved once it is shown that the two remaining integrals on the right hand side of (2.7) vanish in the limit $r_\pm \to \infty$.

3) The remaining step in the proof is therefore to show

$$\lim_{r_\pm \to \infty} \int_0^{r_\pm} \tau_{\mu\nu}(h_\pm(r_\pm,\rho)) \, d\rho = 0 \,. \tag{2.8}$$

We will demonstrate this only for the "+" case, the reasoning for the "−" case is similar.

It holds that $B \in \mathcal{A}_\infty(\mathcal{O}_I)$ for $I = \{x^1 e_1 : |x^1| < \sqrt{2}\,\xi\}$ with some sufficiently large $\xi > 0$. By Prop. 2.3(d), $\tau_{\mu\nu}(x) = 0$ for $x \in (\mathcal{O}_I)^\perp$, implying that

$$\int_0^{r_+} \tau_{\mu\nu}(h_+(r_+,\rho)) \, d\rho = \int_0^{\xi} \tau_{\mu\nu}(h_+(r_+,\rho)) \, d\rho \,, \tag{2.9}$$

i.e. the integral extends for all $r_+ > 0$ only over a fixed interval of finite length.

Now choose some wedge region W in the causal complement of $\bigcup_{r_+ \geq 0} H_+(r_+) \subset W_R$, and let $\delta > 0$ be arbitrary. According to (2.3), one can find some $B_\delta \in \mathcal{A}_\infty(W)$ so that

$$|\langle A\Omega, T_{\mu\nu}[x](B - B_\delta)\Omega\rangle| < \frac{\delta}{2\xi}$$

uniformly in $x \in \mathbb{R}^2$. Then $\langle A\Omega, [T_{\mu\nu}[x], B_\delta]\Omega\rangle = 0$ for all $x \in H_+(r_+)$, and

$$\int_0^{\xi} \tau_{\mu\nu}(h_+(r_+,\rho)) \, d\rho = \int_0^{\xi} \langle A\Omega, T_{\mu\nu}[h_+(r_+,\rho)](B - B_\delta)\Omega\rangle \, d\rho$$
$$+ \int_0^{\xi} \langle (B_\delta^* - B^*)A\Omega, T_{\mu\nu}[h_+(r_+,\rho)]\Omega\rangle \, d\rho \,.$$

The absolute value of the first integral on the right hand side of the last equation can be estimated by $\xi \cdot \delta/2\xi = \delta/2$. Owing to (2.4), the other integral on the right hand side of the last equation converges to 0 for $r_+ \to \infty$ (note that the integrands are bounded uniformly in r_+). Therefore we can find for the given $\delta > 0$ some $r > 0$ so that $|\int_0^\xi \tau_{\mu\nu}(h_+(r_+,\rho)) \, d\rho| < \delta$ for all $r_+ > r$. By (2.9), this establishes the required relation (2.8), and thus the proof is complete. $\square$

3 A result for lightlike half-lines

In this section we present a result indicating that ANEC fails to hold in general for dense subsets of the vectors considered in Theorem 2.5 when the expectation value of the energy-momentum tensor is integrated only over a lightlike half-line. This is of course no surprise in view of the fact that a lightlike half-line has a large causal complement together with the assumed properties of the energy-momentum tensor. The precise formulation of the result is as follows.

Proposition 3.1. *Let $\{\mathcal{H}, \mathcal{A}, U, \Omega, T_{\mu\nu}\}$ be a quantum field theory with energy-momentum tensor on two-dimensional Minkowski-spacetime with the properties assumed in Section 2.1. Let k be a non-zero lightlike vector in $\mathbb{R}^2$, $a \in \mathbb{R}^2$, and let $\mathcal{O}$ be a double cone lying in the causal complement of the lightlike half-line $L := \{s \cdot k + a : s \geq 0\}$.*

Suppose that $\mathcal{A}_\infty(\mathcal{O})\Omega$ is dense in $\mathcal{H}$ (Reeh-Schlieder property) and that for all $A \in \mathcal{A}_\infty(\mathcal{O})$ there holds

$$\liminf_{r\to\infty} \int_0^r \langle A\Omega, T_{\mu\nu}[s\cdot k + a]A\Omega\rangle k^\mu k^\nu \, ds \geq 0 \, .$$

Then the Hilbertspace $\mathcal{H}$ is one-dimensional and spanned by the vacuum vector Ω, and $T_{\mu\nu}(f) = 0$ for all $f \in C_0^\infty(\mathbb{R}^2)$.

Proof. We consider only the case $k = e_+$ and $a = 0$, the general case is proved analogously.

Then we observe that

$$\langle A\Omega, T[L]B\Omega\rangle := \lim_{r\to\infty} \int_0^r \langle A\Omega, T_{\mu\nu}[s\cdot e_+]A\Omega\rangle e_+^\mu e_+^\nu \, ds = i\langle A\Omega, T_{\mu\nu}[0]B\Omega\rangle e_+^\mu e_+^\nu \tag{3.1}$$

holds for all $A, B \in \mathcal{A}_\infty(\mathcal{O})$ as can be seen from (2.5) together with the fact that $\langle A\Omega, [T_{\mu\nu}[s\cdot e_+], B]\Omega\rangle = 0$, $s \geq 0$. Equation (3.1) defines a quadratic form $\langle \, . \, , T[L] \, . \, \rangle$ on $\mathcal{A}_\infty(\mathcal{O})\Omega \times \mathcal{A}_\infty(\mathcal{O})\Omega$ which is by assumption positive, i.e. $\langle A\Omega, T[L]A\Omega\rangle \geq 0$, $A \in \mathcal{A}_\infty(\mathcal{O})$. It follows that there is an essentially selfadjoint, positive operator $T_L^{1/2}$ with domain $\mathcal{A}_\infty(\mathcal{O})\Omega$ so that

$$\langle T_L^{1/2}A\Omega, T_L^{1/2}B\Omega\rangle = \langle A\Omega, T[L]B\Omega\rangle = \langle B^*A\Omega, T[L]\Omega\rangle \, , \quad A, B \in \mathcal{A}_\infty(\mathcal{O}) \, .$$

Using $\langle \Omega, T[L]\Omega\rangle = 0$ this implies $\langle A\Omega, T[L]B\Omega\rangle = 0$ and hence, by (3.1),

$$\begin{aligned} &\langle (1+P_0)^\ell A\Omega, (1+P_0)^{-\ell}T_{\mu\nu}[0](1+P_0)^{-\ell}(1+P_0)^\ell B\Omega\rangle e_+^\mu e_+^\nu \\ &= \ \langle A\Omega, T_{\mu\nu}[0]B\Omega\rangle e_+^\mu e_+^\nu \ = \ -i\langle A\Omega, T[L]B\Omega\rangle \ = \ 0 \end{aligned}$$

for all $A, B \in \mathcal{A}_\infty(\mathcal{O})$. The set of vectors $(1+P_0)^\ell A\Omega$, $A \in \mathcal{A}_\infty(\mathcal{O})$ is dense in $\mathcal{H}$, therefore, using also covariance (Prop. 2.3(b)), one arrives at

$$(1+P_0)^{-\ell}T_{\mu\nu}[x](1+P_0)^{-\ell}e_+^\mu e_+^\nu = 0 \, , \quad x \in \mathbb{R}^2 \, .$$

Thus $\langle A\Omega, T_{\mu\nu}[x]B\Omega\rangle e_+^\mu e_+^\nu = 0$ for all $A, B \in \mathcal{A}_\infty$ and $x \in \mathbb{R}^2$, and in view of Theorem 2.5, this entails

$$(P_0 + P_1)B\Omega = 0\,, \quad B \in \mathcal{A}_\infty\,.$$

Since we have imposed the mass gap assumption (iv), we may apply Proposition I.1.2 of [6] to conclude that this is only possible if $B\Omega$ is parallel to the vacuum vector Ω. As the set of vectors $\mathcal{A}_\infty\Omega$ is dense in $\mathcal{H}$, this implies $\mathcal{H} = \mathbb{C}\cdot\Omega$, and by the vanishing of the vacuum-expectation value of $T_{\mu\nu}$, finally $T_{\mu\nu}(f) = 0$ for all $f \in C_0^\infty(\mathbb{R}^2)$. $\square$

4 Concluding remarks

It has been shown that in two-dimensional Minkowski spacetime the ANEC can be derived under very general hypotheses for quantum field theories endowed with an energy-momentum tensor. The two-dimensionality was quite essential in exploiting the vanishing of the divergence of $T_{\mu\nu}$ in the proof of Thm. 2.5 and it is not at all clear if our simple argument can be generalized to higher dimensions. So the general validity of ANEC in higher dimensions remains unsettled.

Concerning quantum field theory in curved spacetime, a familiar problem is that there are no candidates for a vacuum state of a quantum field theory owing to the circumstance that in general there are no spacetime symmetries. Then already the characterization of physical states and the definition of expectation values of an energy-momentum tensor poses considerable problems (see [22] for discussion how this problem is treated in the case of free fields). Clearly the the question if, and in which sense, ANEC may hold in quantum field theory in curved spacetime is connected to this circle of problems, particularly to the issue of how to characterize states which may be viewed as playing the role of preferred, vacuum-like states. Here ANEC is in some ways attractive as imposing a global constraint on candidates for such states, complementary to other prominent conditions, like the Hadamard condition (cf. [22]) or the microlocal spectrum condition (see [4]) which constrain the short-distance properties of physical states. A drawback is that ANEC is a condition which cannot be tested locally. It is to be hoped that more progress in understanding the relation between the said local conditions and ANEC will be made in the future.

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